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A Sort of New Improved Algorithm For Total Least Square

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Abstract: Aim to blemish of total least square algorithm based on error equation of virtual observation, this paper put forward and deduced a sort of new improved algorithm which selects essential unknown parameters among designing matrix, and then, doesn't consider condition equation of unknown parameters among designing matrix. So, this paper perfected and enriched algorithm, and sometimes, new method of this paper is better. Finally, the results of examples showed that new mothod is viable and valid.

Keywords: Error equation, essential unknown parameters, mean square error of unit weight, total least square, virtual observation.

1. INTRODUCTION

Literatures [1, 2] made some reasonable improvement such as putting forward virtual observation method and listing error equations of virtual observation for total least square algorithm, but there existed some problem. Aim to blemish of total least square algorithm based on literatures [1, 2], literatures [3] has put forward and deduced further improved algorithm which considered condition equation of unknown parameters among designing matrix. But some times, the improved algorithm isn't always better, so, this paper proposed and deduced a sort of new algorithm which selects essential unknown parameters among designing matrix, and then, doesn't consider condition equation of unknown parameters among designing matrix, perfects and enriches algorithm of total least square algorithm. Finally, imitative example showed that new method is viable and valid.

2. THE PRINCIPLE OF IMPROVED ALGORITHM OF TOTAL LEAST SQUARE

2.1. Error Equations of Actual Observations

We suppose there exists function model of adjustment of indirect observations based on actual observation [1-3]

$$\underset{n,1}{\hat{L}} = \underset{n,t}{\hat{B}}\,\underset{t,1}{\hat{X}} - \underset{n,1}{d} \quad , \quad \underset{n,n}{P_L} \tag{1}$$

where,

$$\underset{n,1}{\hat{L}} = \begin{bmatrix} \hat{L}_1 \\ \hat{L}_2 \\ \vdots \\ \hat{L}_n \end{bmatrix}, \underset{n,t}{\hat{B}} = \begin{bmatrix} \hat{B}_{11} & \hat{B}_{12} & \cdots & \hat{B}_{1t} \\ \hat{B}_{21} & \hat{B}_{22} & \cdots & \hat{B}_{2t} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{B}_{n1} & \hat{B}_{n2} & \cdots & \hat{B}_{nt} \end{bmatrix},$$

$$\underset{t,1}{\hat{X}} = \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \vdots \\ \hat{X}_t \end{bmatrix}, \underset{n,1}{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}.$$

n--- quantity of actual observations;

t--- necessary quantity of unknown parameters;

$\underset{n,1}{\hat{L}}$ --- estimated value of column vector of observations;

$\underset{n,t}{\hat{B}}$ --- estimated value of designing matrix(part of $\hat{B}$ may be constant and haven't error);

$\underset{t,1}{\hat{X}}$ ---estimated value of column vector of unknown parameters;

$\underset{n,1}{d}$ --- column vector of constant;

$\underset{n,n}{P_L}$ --- weight matrix of actual observations.

We suppose any element of $\underset{n,t}{\hat{B}}$ can be expressed by unknown parameters

$\underset{t',1}{\hat{Y}} = \left(\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_{t'}\right)^T$.

There, $\hat{Y}_i$ and $\hat{Y}_j$ ($1 \le i, j \le t'$ and $i \ne j$) are independent.Then,based on Eq.1,we can obtain

$$\underset{n,1}{V_L} = \begin{bmatrix} \underset{n,t}{B^0} & \underset{n,t'}{A} \end{bmatrix} \begin{bmatrix} \underset{t,1}{\hat{x}} \\ \underset{t',1}{\hat{y}} \end{bmatrix} - (\underset{n,1}{d} + \underset{n,1}{L} - \underset{n,t}{B^0}\,\underset{t,1}{X^0}) \quad , \quad \underset{n,n}{P_L} \tag{2}$$

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where:

$\underset{n,1}{L}$ ---actual observations;

$V\underset{n,1}{L}$ --- residuals of observations $\underset{n,1}{L}$;

$\underset{n,1}{\hat{L}} = V\underset{n,1}{L} + \underset{n,1}{L}$;

$\underset{n,t}{B^0}$ --- approximations of $\underset{n,t}{\hat{B}}$;

$\underset{n,t}{\Delta\hat{B}}$ --- residuals of $\underset{n,t}{B^0}$;

$\underset{n,t}{\hat{B}} = \underset{n,t}{B^0} + \underset{n,t}{\Delta B}$;

t' --- necessary quantity of unknown parameters of element of $\underset{n,t}{\hat{B}}$;

$\underset{t',1}{Y^0} = \left(Y_1^0, Y_2^0, \cdots, Y_{t'}^0\right)^T$ ---approximations of $\underset{t',1}{\hat{Y}} = \left(\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_{t'}\right)^T$;

$\underset{t',1}{\hat{y}} = \left(\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_{t'}\right)^T$ --- residuals of $\underset{t',1}{Y^0} = \left(Y_1^0, Y_2^0, \cdots, Y_{t'}^0\right)^T$;

$\underset{t',1}{\hat{Y}} = \underset{t',1}{Y^0} + \underset{t',1}{\hat{y}}$;

$\underset{t,1}{X^0} = \left(X_1^0, X_2^0, \cdots, X_t^0\right)^T$ ---approximations of $\underset{t,1}{\hat{X}} = \left(\hat{X}_1, \hat{X}_2, \cdots, \hat{X}_t\right)^T$;

$\underset{t,1}{\hat{x}} = \left(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_t\right)^T$ --- residuals of $\underset{t,1}{X^0} = \left(X_1^0, X_2^0, \cdots, X_t^0\right)^T$;

$\underset{t',1}{\hat{X}} = \underset{t',1}{X^0} + \underset{t',1}{\hat{x}}$;

$\underset{n,t'}{A}$ --- coming from $\underset{n,t'}{A}\underset{t',1}{\hat{Y}} = \underset{n,t}{\Delta\hat{B}}\underset{t,1}{X^0}$.

In Eq. 2, we don't consider $\underset{n,t}{\Delta\hat{B}}\underset{t,1}{\hat{x}}$ which is very small.

Obviously, Eq. 2 of this paper is different from homologous equations of literatures [1-3].

2.2. Error Equations of Virtual Observations

Because we suppose any element of $\underset{n,t}{\hat{B}}$ can be expressed by unknown parameters

$\underset{t',1}{\hat{Y}} = \left(\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_{t'}\right)^T$, and $\hat{Y}_i$ and $\hat{Y}_j$ $(1 \le i, j \le t'$ and $i \ne j)$ are independent, so, if $\hat{B}_{ij}$ hasn't error,we don't list error equation;if $\hat{B}_{ij}$ has error, we list error equation

$$V_{B_{ij}} = \underset{1,t'}{f_{ij}}\underset{t',1}{\hat{y}} - (B_{ij} - B_{ij}^0), \quad P_{B_{ij}} \tag{3}$$

where:

B_{ij} --- observations of $\hat{B}_{ij}$;

B_{ij}^0 --- approximations of $\hat{B}_{ij}$;

$V_{B_{ij}}$ --- residuals of B_{ij};

$P_{B_{ij}}$ --- weight matrix of virtual observation B_{ij};

$f_{ijk} = \dfrac{\partial \hat{B}_{ij}}{\partial \hat{y}_k}\Big|\hat{B}_{ij} = B_{ij}^0$, $k = 1, 2, \cdots, t'$;

$\underset{1,t'}{f_{ij}} = \begin{pmatrix} f_{ij1} & f_{ij2} & \cdots & f_{ijt'} \end{pmatrix}$.

Based on Eq.3,We can obtain

$$\underset{k,1}{V_B} = \underset{k,t'}{f}\underset{t',1}{\hat{y}} - \underset{k,1}{l_B}, \quad \underset{k,k}{P_B} \tag{4}$$

where:

k---quantity of virtual observation($0 \le k \le nt$);

$\underset{k,k}{P_B}$ --- weight matrix of virtual observations;

$$\underset{k,1}{V_B} = \begin{bmatrix} V_{B11} \\ V_{B21} \\ \vdots \\ V_{Bn,1} \\ V_{B12} \\ V_{B22} \\ \vdots \\ V_{Bn2} \\ \vdots \\ V_{Bnt} \end{bmatrix}, \quad \underset{k,1}{l_B} \begin{bmatrix} B_{11} - B_{11}^0 \\ B_{21} - B_{21}^0 \\ \vdots \\ B_{n1} - B_{n1}^0 \\ B_{12} - B_{12}^0 \\ B_{22} - B_{22}^0 \\ \vdots \\ B_{n2} - B_{n2}^0 \\ \vdots \\ B_{nt} - B_{nt}^0 \end{bmatrix}, \quad \underset{kt'}{f} = \begin{bmatrix} \underset{1t'}{f_{11}} \\ \underset{1t'}{f_{21}} \\ \vdots \\ \underset{1t'}{f_{n1}} \\ \underset{1t'}{f_{12}} \\ \underset{1t'}{f_{22}} \\ \vdots \\ \underset{1t'}{f_{n2}} \\ \vdots \\ \underset{1t'}{f_{n,t}} \end{bmatrix}$$

If $\hat{B}_{ij}$ hasn't error, we should delete homologous elements of $\underset{k,1}{V_B}$, $\underset{k,t'}{f}$ and $\underset{k,1}{l_B}$ in Eq.4 ,and homologous elements of $\underset{k,k}{P_B}$.

Obviously, Eq. 4 of this paper is still different from homologous equations of literatures [1-3].

2.3. Total Error Equations

Based on Eq. 2 and Eq. 4,we can obtain total error equations

$$\underset{n+k,1}{V} = \underset{n+k,t+t'}{C}\ \underset{t+t',1}{\hat{z}} - \underset{n+k,1}{l}, P \tag{5}$$

where:

$$\underset{n+k,1}{V} = \begin{bmatrix} \underset{n,1}{V_L} \\ \underset{k,1}{V_B} \end{bmatrix}, \quad \underset{n+k,t+t'}{C} = \begin{bmatrix} \underset{n,t}{B^0} & \underset{n,t'}{A} \\ \underset{k,t}{0} & \underset{k,t'}{f} \end{bmatrix}, \quad \underset{t+t',1}{\hat{z}} = \begin{bmatrix} \underset{t,1}{\hat{x}} \\ \underset{t',1}{\hat{y}} \end{bmatrix},$$

$$\underset{n+k,1}{l} = \begin{bmatrix} \underset{n,1}{d} + \underset{n,1}{L} - \underset{n,t}{B^0}\ \underset{t,1}{X^0} \\ \underset{k,1}{l_B} \end{bmatrix}.$$

P--- total weight matrix;

$\underset{k,\ t}{0}$ of left bottom of $\begin{bmatrix} \underset{n,t}{B^0} & \underset{n,t'}{A} \\ \underset{k,t}{0} & \underset{k,t'}{f} \end{bmatrix}$ is zero matrix.

Obviously, Eq. 5 of this paper is different from homologous equations of literatures [1-3].

2.4. Solving out Total Error Equations [4]

Based on least square method and Eq. 5, we can obtain estimated values of unknown parameters

$$\underset{t+t',1}{\hat{z}} = \left(\underset{t+t',n+k}{C^T}\ \underset{n+k,n+k}{P}\ \underset{n+k,t+t'}{C} \right)^{-1} \underset{t+t',n+k}{C^T}\ \underset{n+k,n+k}{P}\ \underset{n+k,1}{l} \tag{6}$$

estimated value of mean square error of unit weight

$$\hat{\sigma}_0 = \pm\sqrt{\frac{\underset{1,n+k}{V^T}\ \underset{n+k,n+k}{P}\ \underset{n+k,1}{V}}{(n+\mathrm{k})-(t+\mathrm{t}')}} \tag{7}$$

and estimated values of variances and covariance of unknown parameters

$$\underset{t+t',t+t'}{D_{\hat{z}\hat{z}}} = \hat{\sigma}_0^{\,2} \left(\underset{t+t',n+k}{C^T}\ \underset{n+k,n+k}{P}\ \underset{n+k,t+t'}{C} \right)^{-1} \tag{8}$$

In this paper, $\hat{B}_{ij}$ may come from $\underset{t',1}{\hat{Y}}$ or $\underset{k,1}{V_B}$, and results of the two method are not same, though the final alternate results are very small. In order to data consistence of estimated value of designing matrix, this paper selected that $\hat{B}_{ij}$ came from $\underset{t',1}{\hat{Y}}$.

Obviously, Eq. 6, Eq. 7 and Eq. 8 of this paper are different from homologous equations of literatures [1-3].

3. FIRST IMITATIVE EXAMPLE

Suppose there existed a plane curve such as

$$\hat{Y}_i = \hat{a}_0 + \hat{a}_1 \hat{X}_i + \hat{a}_2 \hat{X}_i^2 + \hat{a}_3 \hat{X}_i^3 \quad i = 1,2,\cdots,n \tag{9}$$

where:

$\hat{a}_0$, $\hat{a}_1 \hat{a}_2$ and $\hat{a}_3$--- estimated values of unknown coefficient;

X_i and Y_i--- measured data with error;

$\hat{X}_i$ and $\hat{Y}_i$ --- estimated value of X_i and Y_i.

and we obtained coordinations of 10 points such as Table 1 based on imitative example.

Table 1. Measured data of plane curve.

Point Name	X (m)	Y (m)
N0.1	3.536	125.145
N0.2	7.319	958.067
N0.3	1.527	15.860
N0.4	4.311	214.911
N0.5	5.473	418.513
N0.6	8.314	1384.036
N0.7	10.639	2834.032
N0.8	2.877	72.382
N0.9	13.521	5712.836
N0.10	15.392	8369.751

3.1. Error Equations of Actual Observations

Based on Eq. 9, we can obtain error equation

$$\underset{10,1}{V_Y} = \underset{10,4}{\hat{B}} \begin{pmatrix} \hat{a}_0 & \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \end{pmatrix}^T - \underset{10,1}{Y}, \underset{10\times 10}{P_Y} \tag{10}$$

where:

$$\underset{10,1}{V_Y} = \begin{bmatrix} V_{Y_1} \\ V_{Y_2} \\ \vdots \\ V_{Y_{10}} \end{bmatrix}, \quad \underset{10,4}{\hat{B}} = \begin{bmatrix} 1 & \hat{X}_1 & \hat{X}_1^2 & \hat{X}_1^3 \\ 1 & \hat{X}_2 & \hat{X}_2^2 & \hat{X}_2^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{X}_n & \hat{X}_n^2 & \hat{X}_n^3 \end{bmatrix}, \quad \underset{10,1}{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{10} \end{bmatrix}$$

V_{Y_i} $(i = 1,2,\cdots,10)$ --- residuals of Y_i $(i = 1,2,\cdots,10)$;

$\underset{10,10}{P_Y}$ --- weight matrix of actual observations.

Element of $\underset{10,4}{\hat{B}}$ can be expressed by unknown parameters $\underset{10,1}{\hat{X}} = \left(\hat{X}_1, \hat{X}_2, \cdots, \hat{X}_{10}\right)^T$, and suppose $\hat{X}_i$ and $\hat{X}_j$ $(1 \le i, j \le 10$ and $i \ne j)$ are independent.Then, we can obtain error equations of actual observations

$$\underset{10,1}{V_Y}=\begin{bmatrix}\underset{10,4}{B_0} & \underset{10,10}{A}\end{bmatrix}\underset{14,1}{\hat{z}}-\underset{10,1}{l_Y}\ ,\ \underset{10,10}{P_Y} \tag{11}$$

where:

$$\underset{10,4}{B}^0=\begin{bmatrix}1 & X_1^0 & \left(X_1^0\right)^2 & \left(X_1^0\right)^3\\ 1 & X_2^0 & \left(X_2^0\right)^2 & \left(X_2^0\right)^3\\ \vdots & \vdots & \vdots & \vdots\\ 1 & X_{10}^0 & \left(X_{10}^0\right)^2 & \left(X_{10}^0\right)^3\end{bmatrix};$$

$\underset{10,10}{A}=diag(\ A_1\ \ A_2\ \ \cdots\ \ A_{10}\ )$;

$\underset{14,1}{\hat{z}}=\begin{pmatrix}\delta\hat{a}_0 & \delta\hat{a}_1 & \delta\hat{a}_2 & \delta\hat{a}_3 & \hat{x}_1 & \hat{x}_2 & \cdots & \hat{x}_{10}\end{pmatrix}^T$;

$\underset{10,1}{l_Y}=\left(\underset{10,1}{Y}-\underset{10,4}{B^0}\underset{4,1}{a^0}\right)$;

$X_i^0(i=1,2,\cdots,10)$ ---approximations of $\hat{X}_i(i=1,2,\cdots,10)$;

$\hat{x}_i(i=1,2,\cdots,10)$ --- residuals of $X_i^0(i=1,2,\cdots,10)$;

$\hat{X}_i=X_i^0+\hat{x}_i(i=1,2,\cdots,10)$;

$A_i=a_1^0+2a_2^0X_i^0+3a_3^0\left(X_i^0\right)^2,(i=1,2,\cdots,10)$;

$a_i^0(i=0,1,2,3)$ ---approximations of $\hat{a}_i(i=0,1,2,3)$;

$\delta\hat{a}_i(i=0,1,2,3)$ --- residuals of $a_i^0(i=0,1,2,3)$;

$\hat{a}_i=a_i^0+\delta\hat{a}_i(i=0,1,2,3)$;

$\underset{4,1}{a}^0=\begin{pmatrix}a_0^0 & a_1^0 & a_2^0 & a_3^0\end{pmatrix}^T.$

3.2. Error Equations of Virtual Observations

Because $\hat{B}_{i1}=1(i=1,2,\cdots,10)$ of $\underset{10,4}{\hat{B}}$ hasn't error,we don't list error equation of virtual observation;and because others of $\underset{10,4}{\hat{B}}$ have error, we should list error equations of virtual observations.

Because element of $\underset{10,4}{\hat{B}}$ can be expressed by parameters $\underset{10,1}{\hat{X}}=\left(\hat{X}_1,\hat{X}_2,\cdots,\hat{X}_{10}\right)^T$, suppose $\hat{X}_i$ and $\hat{X}_j$ $(1\le i,j\le 10$ and $i\neq j)$ are independent,we can obtain error equations of virtual observations

$$\underset{30,1}{V_B}=\underset{30,10}{f}\ \underset{10,1}{\hat{x}}-\underset{30,1}{l_B}\ ,\ \underset{30,30}{P_B} \tag{12}$$

where:

$\underset{30,1}{V_B}=\begin{pmatrix}V_{B12} & V_{B22} & \cdots & V_{B102} & V_{B13} & V_{B23} & \cdots & V_{B103} & \cdots & V_{B104}\end{pmatrix}^T$

$$\underset{30,10}{f}=\begin{bmatrix}1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1\\ 2X_1^0 & & & \\ & 2X_2^0 & & \\ & & \ddots & \\ & & & 2X_{10}^0\\ 3\left(X_1^0\right)^2 & & & \\ & 3\left(X_2^0\right)^2 & & \\ & & \ddots & \\ & & & 3\left(X_{10}^0\right)^2\end{bmatrix},$$

$\underset{10,1}{\hat{x}}=\begin{pmatrix}\hat{x}_1 & \hat{x}_2 & \cdots & \hat{x}_{10}\end{pmatrix}^T$,

$\underset{30,1}{l_B}=\begin{pmatrix}B_{12}-B_{12}^0 & B_{22}-B_{22}^0 & \cdots & B_{102}-B_{102}^0 & B_{13}-B_{13}^0 & \cdots & B_{104}-B_{104}^0\end{pmatrix}^T$,

$\underset{30,30}{P_B}$ --- weight matrix of virtual observations.

3.3. Total Error Equations

Based on Eq. 11 and Eq. 12, we can obtain total error equations

$$\underset{40,1}{V}=\underset{40,14}{C}\ \underset{14,1}{\hat{z}}-\underset{40,1}{l}\ ,\ \underset{40,40}{P} \tag{13}$$

where:

$$\underset{40,1}{V}=\begin{bmatrix}\underset{10,1}{V_Y}\\ \underset{30,1}{V_B}\end{bmatrix},\ \underset{40,14}{C}=\begin{bmatrix}\underset{10,4}{B^0} & \underset{10,10}{A}\\ \underset{30,4}{0} & \underset{30,10}{f}\end{bmatrix},\ \underset{40,1}{l}=\begin{bmatrix}\underset{10,1}{l_Y}\\ \underset{30,1}{l_B}\end{bmatrix},$$

$\underset{30,4}{0}$ of left bottom of $\begin{bmatrix}\underset{10,4}{B^0} & \underset{10,10}{A}\\ \underset{30,4}{0} & \underset{30,10}{f}\end{bmatrix}$ is zero matrix,

$\underset{40,40}{P}$ --- total weight matrix.

3.4. Solving out Total Error Equations

Based on least square method and Eq.13, we can obtain estimated values of unknown parameters

$$\underset{14,1}{\hat{z}}=\left(\underset{14,40}{C}^T\underset{40,40}{P}\underset{40,14}{C}\right)^{-1}\underset{14,40}{C}^T\underset{40,40}{P}\underset{40,1}{l} \tag{14}$$

estimated value of mean square error of unit weight

Table 2. Iterative Computing $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.257497982	± 0.25736601	± 0.257366001	± 0.257366001
$\hat{a}_0$	-0.4282326	-0.47119948	-0.47160641	-0.47161343
$\hat{a}_1$	2.91045167	2.92884547	2.929012661	2.929015863
$\hat{a}_2$	1.51054987	1.508330965	1.508311498	1.508311101
$\hat{a}_3$	2.18464297	2.1847223	2.184722976	2.184722991

Table 3. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$			
first	2.097672	-0.948840	0.117942	-0.004282
	-0.948840	0.472876	-0.061947	0.002321
	0.117942	-0.061947	0.008448	-0.000325
	-0.004282	0.002321	-0.000325	1.3E-05
second	2.185027	-0.985111	0.122220	-0.004433
	-0.985111	0.487931	-0.063721	0.002383
	0.122220	-0.063721	0.008656	-0.000332
	-0.004433	0.002383	-0.000332	1.3E-05
third	2.186504	-0.985701	0.122288	-0.004435
	-0.985701	0.488168	-0.063748	0.002384
	0.122288	-0.063748	0.008660	-0.000333
	-0.004435	0.002384	-0.000332	1.3E-05
fourth	2.186528	-0.985711	0.122290	-0.004435
	-0.985711	0.488172	-0.063749	0.002384
	0.122290	-0.063749	0.008660	-0.000333
	-0.004435	0.002384	-0.000332	1.3E-05

$$\hat{\sigma}_0 = \pm\sqrt{\frac{V^T_{1,40}\,P_{40,40}\,V_{40,1}}{(10+30)-(4+10)}} \tag{15}$$

and estimated values of cofactor matrix of unknown parameters

$$D_{\hat{z}\hat{z}\,14,14} = \hat{\sigma}_0^2\left(C^T_{14,40}\,P_{40,40}\,C_{40,14}\right)^{-1} \tag{16}$$

3.5. Results of Iterative Computing of This paper

If we select No. 1, No. 2, No. 3 and No. 4 of Table **1**, we can obtain approximations of unknown coefficient

$$a^0_{4,1} = \left(\; 1.197125 \quad 2.027197 \quad 1.632403 \quad 2.179726 \;\right)^T$$

In order to simplicity,this paper seleted total weight matrix $P_{40,40}$ as unit weight matrix,then ,we can obtain Table **2** and Table **3** according to new method of this paper,obtain Table **4** and Table **5** according to the method of Literatures [1-2], and obtain Table **6** and Table **7** according to Literatures [3].

Because our aim is to solve out curve, so, in Table **2**, Table **3**, Table **4**, Table **5**, Table **6**, and Table **7**, this paper only listed mean square error of unit weight,coefficients and their variances and covariance.

3.6. Results of Iterative Computing Based on Method of Literatures [1, 2]

Table 4. Iterative Computing $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$.

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.377164774	± 0.306303861	± 0.293593374	± 0.289064237
$\hat{a}_0$	-0.08473767	-2.40450675	-3.29012507	-3.98370589
$\hat{a}_1$	2.750551874	3.950898798	4.412274767	4.775093715
$\hat{a}_2$	1.530753891	1.370558058	1.308644367	1.259795641
$\hat{a}_3$	2.183902004	2.189966546	2.192319246	2.19417964

Table 5. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
first	6.836696	-3.205142	0.406789	-0.014961
	-3.205142	1.668317	-0.223720	0.008497
	0.406789	-0.223720	0.031304	-0.001223
	-0.014961	0.008497	-0.001223	4.9E-05
second	4.747154	-2.181921	0.273889	-0.010007
	-2.181921	1.137142	-0.152644	0.005802
	0.273889	-0.152644	0.021566	-0.000848
	-0.010007	0.005802	-0.000848	3.4E-05
third	9.157135	-4.363607	0.558724	-0.020657
	-4.363607	2.268892	-0.303995	0.011539
	0.558724	-0.303995	0.042199	-0.001640
	-0.020657	0.011539	-0.001640	6.5E-05
fourth	10.780828	-5.159566	0.662257	-0.024521
	-5.159566	2.683191	-0.359551	0.013649
	0.662257	-0.359551	0.049830	-0.001935
	-0.024521	0.013649	-0.001935	7.6E-05

3.7. Results of Iterative Computing Based on Method of Literatures [3]

Table 6. Iterative Computing $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$.

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.257497982	± 0.25736601	± 0.257366001	± 0.257366001
$\hat{a}_0$	-0.4282326	-0.47119948	-0.47160641	-0.47161343
$\hat{a}_1$	2.91045167	2.92884547	2.929012661	2.929015863
$\hat{a}_2$	1.51054987	1.508330965	1.508311498	1.508311101
$\hat{a}_3$	2.18464297	2.1847223	2.184722976	2.184722991

Table 7. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
first	2.097672	-0.948840	0.117942	-0.004282
	-0.948840	0.472876	-0.061947	0.002321
	0.117942	-0.061947	0.008448	-0.000325
	-0.004282	0.002321	-0.000325	1.3E-05
second	2.185027	-0.985111	0.122220	-0.004433
	-0.985111	0.487931	-0.063721	0.002383
	0.122220	-0.063721	0.008656	-0.000332
	-0.004433	0.002383	-0.000332	1.3E-05
third	2.186504	-0.985701	0.122288	-0.004435
	-0.985701	0.488168	-0.063748	0.002384
	0.122288	-0.063748	0.008660	-0.000333
	-0.004435	0.002384	-0.000332	1.3E-05
fourth	2.186528	-0.985711	0.122290	-0.004435
	-0.985711	0.488172	-0.063749	0.002384
	0.122290	-0.063749	0.008660	-0.000333
	-0.004435	0.002384	-0.000332	1.3E-05

3.8. Results of Comparison Analysis

From Table **2**,Table **3**, Table **4**, Table **5**, Table **6**, and Table **7** of this imitative example, we can find out:

(1) New method of this paper is better at algorithm being stable, mean square error of unit weight being small, and matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$ being stable and small than the method of literatures [1, 2].

(2) The results of new method of this paper is almost as same as the results of the method of literature [3].

In a word, this example showed that new mothod of this paper is viable and valid and still supported literature [3].

4. SECOND IMITATIVE EXAMPLE

If Table **1** changes into Table **8**, we can obtain Table **9** and Table **10** according to new method of this paper,obtain Table **11** and Table **12** according to the method of Literatures [1-2], and obtain Table **13** and Table **14** according to Literatures [3].

If we select No. 1, No. 2, No. 3 and No. 4 of Table **8**, we can obtain approximations of unknown coefficient $a_{4,1}^{0}=\begin{pmatrix} 2.360120 & 1.788430 & 0.341557 & 0.725756 \end{pmatrix}^T$, and obtain follow computation.

4.1. Results of Iterative Computing of This paper

Table 8. Measured data of plane curve.

Point name	X (m)	Y (m)
N0.1	1.363	7.270
N0.2	0.596	3.701
N0.3	2.419	18.958
N0.4	1.541	8.583
N0.5	3.125	33.061
N0.6	2.113	14.587
N0.7	1.894	11.981
N0.8	2.151	15.084
N0.9	0.953	4.962
N0.10	1.581	8.951

Table 9. Iterative computing $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$.

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.006406091	± 0.006406885	± 0.006406885	± 0.006406885
$\hat{a}_0$	2.61877099	2.61971445	2.61972163	2.61972169
$\hat{a}_1$	1.03091098	1.03000236	1.02998864	1.02998853
$\hat{a}_2$	0.94419441	0.94439666	0.94440431	0.94440437
$\hat{a}_3$	0.58970185	0.58970223	0.58970094	0.58970094

Table 10. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
first	0.001725	-0.003180	0.001738	-0.000288
	-0.003180	0.006195	-0.003497	0.000593
	0.001738	-0.003497	0.002018	-0.000348
	-0.000288	0.000593	-0.000348	0.000061
second	0.001651	-0.003069	0.001683	-0.000280
	-0.003069	0.006029	-0.003416	0.000580
	0.001683	-0.003416	0.001979	-0.000341
	-0.000280	0.000580	-0.000341	0.000060
third	0.001650	-0.003068	0.001683	-0.000280
	-0.003068	0.006026	-0.003414	0.000580
	0.001683	-0.003414	0.001978	-0.000341
	-0.000280	0.000580	-0.000341	0.000060
fourth	0.001650	-0.003068	0.001683	-0.000280
	-0.003068	0.006026	-0.003414	0.000580
	0.001683	-0.003414	0.001978	-0.000341
	-0.000280	0.000580	-0.000341	0.000060

4.2. Results of Iterative Computing Based on Method of Literatures [1-2]

Table 11. Iterative computing $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$.

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.008408085	± 0.010151230	± 0.010207209	± 0.010210152
$\hat{a}_0$	2.63276735	2.61192737	2.62314999	2.62202865
$\hat{a}_1$	1.00400443	1.04681364	1.02324480	1.02557058
$\hat{a}_2$	0.95924510	0.93430879	0.94825276	0.94688871
$\hat{a}_3$	0.58716728	0.59148182	0.58904186	0.58927905

Table 12. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
first	0.005260	-0.010251	0.005794	-0.000984
	-0.010251	0.021089	-0.012296	0.002129
	0.005794	-0.012296	0.007328	-0.001289
	-0.000984	0.002129	-0.001289	0.000230
second	0.005110	-0.009952	0.005623	-0.000954
	-0.009952	0.020474	-0.011937	0.002067
	0.005623	-0.011937	0.007116	-0.001252
	-0.000954	0.002067	-0.001252	0.000223
third	0.005366	-0.010463	0.005915	-0.001004
	-0.010463	0.021522	-0.012548	0.002172
	0.005915	-0.012548	0.007475	-0.001315
	-0.001004	0.002172	-0.001315	0.000234
fourth	0.005291	-0.010312	0.005829	-0.000989
	-0.010312	0.021213	-0.012367	0.002141
	0.005829	-0.012367	0.007369	-0.001296
	-0.000989	0.002141	-0.001296	0.000231

4.3. Results of Iterative Computing Based on Method of Literatures [3]

Table 13. Iterative computing $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$.

Iterative times	First	Second	Third	Fourth
$\hat{\sigma}_0$	± 0.00640609	± 0.006406885	± 0.006406885	± 0.006406885
$\hat{a}_0$	2.61877099	2.61971445	2.61972163	2.61972169
$\hat{a}_1$	1.03091098	1.03000236	1.02998864	1.02998853
$\hat{a}_2$	0.94419441	0.94439666	0.94440431	0.94440437
$\hat{a}_3$	0.58970185	0.58970223	0.58970094	0.58970094

Table 14. Iterative computing variances and covariance.

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
first	0.001725	-0.003180	0.001738	-0.000288
	-0.003181	0.006195	-0.003497	0.000593
	0.001738	-0.003497	0.002018	-0.000348
	-0.000288	0.000593	-0.000348	0.000061
second	0.001651	-0.003069	0.001683	-0.000280
	-0.003069	0.006029	-0.003416	0.000580
	0.001683	-0.003416	0.001979	-0.000341
	-0.000280	0.000580	-0.000341	0.000060

(Table 14) contd…..

Iterative times	Matrix of variances and covariance of $\hat{a}_0, \hat{a}_1, \hat{a}_2$ and $\hat{a}_3$			
third	0.001650	-0.003068	0.001683	-0.000280
	-0.003068	0.006026	-0.003414	0.000580
	0.001683	-0.003414	0.001978	-0.000341
	-0.000280	0.000580	-0.000341	0.000060
fourth	0.001650	-0.003068	0.001683	-0.000280
	-0.003068	0.006026	-0.003414	0.000580
	0.001683	-0.003414	0.001978	-0.000341
	-0.000280	0.000580	-0.000341	0.000060

4.4. Results of Comparison Analysis

From Table **9**, Table **10**, Table **11**, Table **12**, Table **13**, and Table **14** of second imitative example, we can obtain same results as that of first imitative example.

CONCLUSION

The new method of this paper is more strict in theory, because it thinks elements of designing matrix may have condition equations, and selects essential unknown parameters to express all virtual observation values among designing matrix.

The new method of this paper is another improved method which isn't the same as the method of literature [3], and has been deduced. So, this paper perfected and enriched total least square algorithm in theoretics.

Two examples showed that new mothod of this paper is viable and valid, and still supported and perfected literature [3] in practice.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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