Research on Advanced Analysis Method of Semi-Rigid Steel Frames

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Abstract: The advanced analysis method used to analyze semi-rigid steel frames is introduced, and the method could take account of many factors influencing structure mechanical performance. These factors include: initial imperfect mode of structure and members, geometrical nonlinearity ($P-\Delta$、$P-\delta$ effects), material nonlinearity and semi-rigid connections. The Curved Stability Function (CSF) element is built to take account of initial imperfect state, semi-rigid connections and plastic hinge, and derivatives of the second-order element stiffness matrix to extend the spatial element stiffness matrix. The nonlinearity behavior of semi-rigid connections and plastic hinge are simulated by the spring element. Judgment criteria of plastic hinge and spring stiffness values are determined, as well as of the structure failure. The three semi-rigid steel frames are directly analyzed by the program named NIDA, and the results of NIDA are comparatively analyzed with test results. It is indicated that the results of NIDA are better compared to the test results, verifying validity and reliability of the advanced analysis method used to analyze semi-rigid steel frames.

Keywords: Curved stability function element, initial imperfect, plastic hinge, semi-rigid steel frames, the advanced analysis method.

1. INTRODUCTION

Steel structures analysis used the first order elastic method to calculate the current steel structure code [1]. The geometric nonlinearity of structures is considered by the calculation length coefficient, and the material nonlinearity of members is considered by the stability factor in design stage. The calculation results by the first order elastic method meet the demand of actual stress and actual displacement of steel structures by calculation coefficient and stability factor. In fact, the actual stress and actual displacement are achieved by revising the results of first order elastic method. The current steel structures design method have much shortcomings as follows: 1) The calculation mode of structures internal force is inconsistent with the calculation mode of member bearing capacity; 2) The conception of calculation length coefficient does not describe correlation of the stability capacity between structures and members; 3) The different structures have different reliability levels of the structures capacity limit state. Many scholars strived to improve the current methods to be applied in elastic stage [2, 3], but those improved methods are essentially the first order elastic analysis method.

In the past decade, professor Chan presented the PEP element [4] considering that the initial imperfect mode of members, computational efficiency and computational accuracy are improved. The nonlinearity computation program named NIDA software developed by professor Chan, used advanced analysis of steel structures. The advanced analysis method considers many influencing factors of structures stress, including initial imperfect mode of members and structures, geometric nonlinearity, material nonlinearity and semi-rigid connections. The advanced analysis method is actually the second order elastic-plastic analysis method. The calculation length coefficient of members is 1.0 with the advanced method. Supposing initial imperfect state is half sine wave shape, a new curved stability function element is introduced in this paper, which is used in advanced method. Also, the second order tangent stiffness matrix is defined in this paper. The spring element is given to simulate semi-rigid connections and plastic hinge. Criteria for plastic hinge and structures failure are defined. At last, the correctness and reliability of the advanced analysis method are verified by semi-rigid connections steel frame test data.

2. SEMI-RIGID CONNECTIONS MODEL

Many scholars proposed semi-rigid connection models, including multiple linear model, polynomial model, spline curve B model, power function model and exponent model. The three parameter powers model is used to simulate the moment and rotation curves of semi-rigid connections [5].

$$M=\frac{R_{ki}\theta}{\left[1+(\theta/\theta_0)^{\mathrm{n}}\right]^{1/\mathrm{n}}} \tag{1}$$

where, R_{ki} is initial stiffness of the connections, $\theta_0 = M_{\mathrm{u}}/R_{\mathrm{ki}}$ is a reference plastic rotation, M_{u} is ultimate moment capacity and n is shape parameter. In this paper, values of the initial stiffness and ultimate moment capacity are acquired by the test data, and the shape parameter, by numerical fitting.

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3. INITIAL IMPERFECT MODE

3.1. Initial Imperfect Mode of Members

The initial imperfect mode of members including initial geometry imperfect and residual stress are simulated by half sine wave [6]; the Fig. (**1**) shows the half sine wave. The representative value of initial imperfect mode is acquired by Eq. (2).

$$\delta_0 = e_0 sin\frac{\pi x}{l} \tag{2}$$

where δ_0 is the initial deformation value, x is the distance from end of members, e_0 is the initial deformation value of middle members, is the x is distance from end of members, l is the length of members, e_0/l is the synthetical representation of the value of initial imperfect, and Table **1** gives the value.

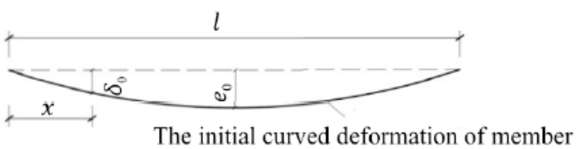

Fig. (1). The member initial imperfect state.

3.2. Initial Imperfect Geometry of Steel Frame

The initial imperfect geometry of steel frame is shown in Fig. (**2**) [7], the representative value of initial imperfect geometry is shown by the Eq. (3) below:

$$\Delta_i = \frac{h_i}{250}\sqrt{0.2+\frac{1}{n_s}}\,\frac{1}{\varepsilon_s} \tag{3}$$

where Δ_i is the value of initial imperfect geometry of calculated storey, h_i is height of the calculated storey, n_s is total number of layers, when $\sqrt{0.2+1/n_s} < 2/3$, $\sqrt{0.2+1/n_s}$=2/3, and when $\sqrt{0.2+1/n_s} > 1$, $\sqrt{0.2+1/n_s} = 1.0$, ε_s is the correction factor of steel grade, $\varepsilon_s = \sqrt{235/f_y}$.

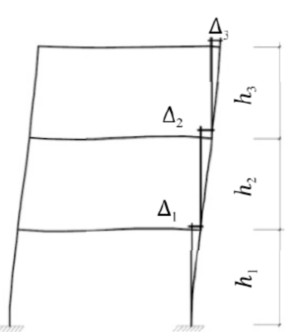

Fig. (2). Initial imperfect geometry of frames.

4. THE CURVED STABILITY FUNCTION ELEMENT

4.1. Assumptions of Element

1. The element is derived by Timoshenko beam-column theory;
2. Only consider small train and large deformation;
3. Do not consider warping effects and shear deformation;
4. Do not consider flexural torsional buckling of members;
5. The initial imperfection of element is simulated by half sine wave; the initial imperfection value is shown in Eq. (2) and Table **1**.

Table 1. The representative value of comprehensive imperfect member.

GB50017 [7] Column Curve	a	b	c	d
e_0/l	1/400	1/350	1/300	1/250

4.2. Deformation Function of Element

The Fig. (**3**) shows element with the initial imperfection.

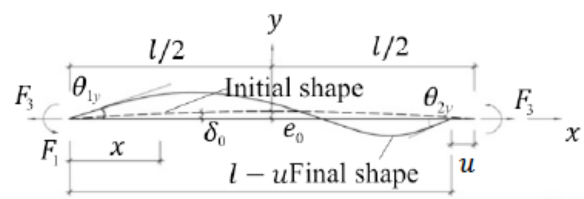

Fig. (3). Calculation diagram of element.

The deformation of element under loading is simulated by the five polynomial formulas, Eq. (4) is given below:

$$v = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 \tag{4}$$

The initial imperfection is acquired by Eq. (2) as follows:

$$\delta_0 = e_0 sin\frac{\pi}{l}\left(x+\frac{l}{2}\right) \tag{5}$$

According to the relation between section moment and section deformation, Eq. (6) is given below:

$$EIv'' = F_3(v+\delta_0) + \frac{F_1+F_2}{l}\left(x+\frac{l}{2}\right) - F_1 \tag{6}$$

According to the boundary conditions, the deformation value of element is calculated by solving the following coefficients, a_0, a_1, a_2, a_3, a_4, a_5.

$$a_0 = \frac{6}{H_2}l(\delta_1-\delta_2) - \frac{q}{H_2}e_0\,, a_1 = -\frac{20}{H_1}(\delta_1-\delta_2)$$

$$a_2 = \frac{48\text{-}q}{2H_2l}(\delta_2-\delta_1) + \frac{8q}{H_2l^2}e_0\,, a_3 = \frac{80\text{-}q}{H_1l^2}(\delta_1+\delta_2)$$

$$a_4 = \frac{2q}{H_2l^3}(\delta_2-\delta_1) - \frac{16q}{H_2l^4}e_0\,, a_5 = \frac{4q}{H_1l^4}(\delta_1+\delta_2)$$

Where $H_1 = 80+q, H_2 = 48+q, q = \frac{F_3l^2}{EI}$

4.3. The Second Order Tangent Stiffness Matrix

Based on the energy principle, the second order stiffness matrix is given below:

$$\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix} = \begin{bmatrix}k_{11} & k_{12} & k_{13}\\ k_{21} & k_{22} & k_{23}\\ k_{31} & k_{32} & k_{33}\end{bmatrix}\begin{bmatrix}\delta_1\\\delta_2\\\delta_3\end{bmatrix} \tag{7}$$

where

$$k_{11}=c_1+c_2+\frac{G_1^2}{H},k_{12}=c_1-c_2+\frac{G_1G_2}{H},k_{13}=\frac{G_1}{lH}$$

$$k_{22}=c_1+c_2+\frac{G_2^2}{H},k_{23}=\frac{G_2}{lH},k_{33}=\frac{1}{l^2H}$$

$$G_1=2b_1(\delta_2+\delta_1)+2b_2(\delta_1-\delta_2)+b_{\mathrm{vs}}\left(\frac{e_0}{l}\right)$$

$$G_2=2b_1(\delta_2+\delta_1)-2b_2(\delta_1-\delta_2)-b_{\mathrm{vs}}\left(\frac{e_0}{l}\right)$$

$$c_1=\frac{19200+800q+8.72q^2+0.02q^3}{H_1^2}$$

$$c_2=\frac{2304+288q+5.8q^2+0.03q^3}{H_2^2}$$

$$H=\frac{1}{\lambda^2}-c_{\mathrm{b}}-c_{\mathrm{b0}},\frac{1}{\lambda^2}=\frac{u/l+c_{\mathrm{b}}+c_{\mathrm{b0}}}{q}$$

$$c_{\mathrm{b}}=b_1(\delta_2+\delta_1)^2+b_2(\delta_1-\delta_2)^2$$

$$c_{\mathrm{b0}}=b_{\mathrm{vs}}\left(\frac{e_0}{l}\right)(\delta_1-\delta_2)+b_{\mathrm{vv}}\left(\frac{e_0}{l}\right)^2$$

$$b_1=\frac{12800+297q+2.19q^2+0.01q^3}{H_1^3}$$

$$b_2=\frac{4608+134q+1.89q^2+0.01q^3}{H_2^3}$$

$$b_{vs}=\frac{7741440+225792q+3168q^2+22q^3}{105H_2^3}$$

$$b_{vv}=-\frac{344064q+9216q^2+64q^3}{35H_2^3}$$

4.4. The Second Order Tangent Stiffness Matrix of Three-Dimensional Element

The Fig. (**4**) shows force at the end of the three-dimensional element.

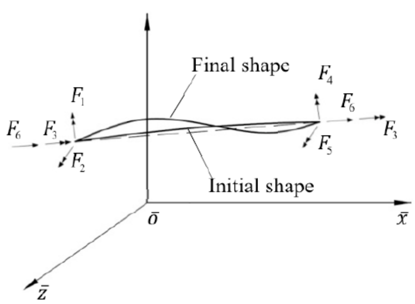

Fig. (4). Three-dimensional diagram of CSF element end force.

According to Eq. (7), the second order tangent matrix of the three-dimensional element is given below:

$$\begin{bmatrix}F_1\\F_2\\F_3\\F_4\\F_5\\F_6\end{bmatrix}=\begin{bmatrix}k_{11}&0&0&k_{14}&0&k_{16}\\0&k_{22}&0&0&k_{25}&k_{26}\\0&0&k_{33}&0&0&0\\k_{41}&0&0&k_{44}&0&k_{46}\\0&k_{52}&0&0&k_{55}&k_{56}\\k_{61}&k_{62}&0&k_{64}&k_{65}&k_{66}\end{bmatrix}\begin{bmatrix}\delta_1\\\delta_2\\\delta_3\\\delta_4\\\delta_5\\\delta_6\end{bmatrix}\tag{8}$$

5. SERI-RIGID CONNECTION

The spring element whose length is zero is put at the end of the element to simulate semi-rigid connections, and the spring stiffness is shown by the moment and rotation curves of three parameter power models.

5.1. The Element Stiffness Matrix with Semi-Rigid Connections

The deformation shape and force at the end of the element is given in Fig. (**5**) below:

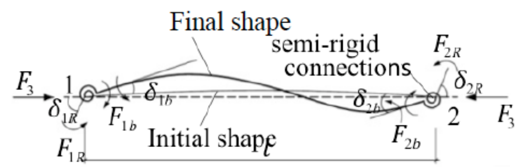

Fig. (5). Diagram of end force and end displacement of CSF element with semi-rigid connections.

The moment and rotation curves are simulated by three parameter power models, and the semi-rigid connections tangent stiffness matrix is given below:

$$[\bar{k}_e]_{6\times6(R)}=\begin{bmatrix}k_{11(R)}&0&0&k_{14(R)}&0&k_{16(R)}\\0&k_{22(R)}&0&0&k_{25(R)}&k_{26(R)}\\0&0&k_{33(R)}&0&0&0\\k_{41(R)}&0&0&k_{44(R)}&0&k_{46(R)}\\0&k_{52(R)}&0&0&k_{55(R)}&k_{56(R)}\\k_{61(R)}&k_{62(R)}&0&k_{64(R)}&k_{65(R)}&k_{66(R)}\end{bmatrix}\tag{9}$$

where,

$$k_{11(\mathrm{R})}=R_1-R_1{}^2(k_{22}+R_2)/\beta_{\mathrm{s}}\,,\ k_{12(\mathrm{R})}=R_1R_2k_{12}/\beta_{\mathrm{s}}$$

$$k_{21(\mathrm{R})}=R_1R_2k_{21}/\beta_{\mathrm{s}}\,,\ k_{22(\mathrm{R})}=R_2-R_2{}^2(k_{11}+R_1)/\beta_{\mathrm{s}}$$

$$k_{13(R)}=k_{31(R)}=k_{31},k_{23(R)}=k_{32(R)}=k_{32},k_{33(R)}=k_{33}$$

$$\beta_{\mathrm{s}}=\begin{vmatrix}k_{11}+R_1&k_{12}\\k_{21}&k_{22}+R_2\end{vmatrix}$$

R_1, R_2 is stiffness of spring at the end of the element.

5.2. Transition Matrix

The six force and displacement of element end is transferred to twelve force and displacement in three dimensional spatial.

$$[\bar{f}_e]_{12\times1(\mathrm{R})}=[T][\bar{f}_e]_{6\times1(\mathrm{R})}\tag{10}$$

$$[\bar{\delta}_e]_{12\times1(\mathrm{R})}=[T][\bar{\delta}_e]_{6\times1(\mathrm{R})}\tag{11}$$

$$[\bar{k}_e]_{12\times12(\mathrm{R})}=[T][\bar{k}_e]_{6\times6(\mathrm{R})}[T]^{\mathrm{T}}\tag{12}$$

6. THE PLASTIC HINGE ELEMENT

The spring element whose length is zero simulate plastic hinge, and changing spring stiffness simulate material non-linearity. When section is yield, the spring element is put in the section, and other section of member is elastic station. When section is yield, the spring stiffness of section R_s is given by Eq. (13) below:

$$R_s = \begin{cases} \infty \ (M \le M_e) \\ \frac{6EI}{l}\frac{|M_P - M|}{|M - M_e|} \ (M_e < M < M_P) \\ 0 \ (M \ge M_P) \end{cases} \quad (13)$$

where M is moment design value of section, M_P is plastic capacity design value of section, M_e is elastic capacity design value of section. When semi-rigid connections and plastic hinge are put at the end of members, the spring element simulating semi-rigid connections and the spring element simulating plastic hinge are put at the end of members. The two spring element stiffness is calculated by the Eq. (14) below:

$$R_{ssi} = \frac{R_{si}R_i}{R_{si}+R_i} \quad (14)$$

where, R_{si} is the spring element simulating plastic hinge, R_i is the spring element simulating semi-rigid connections, R_{ssi} is the total stiffness of the two spring element.

7. THE ADVANCED ANALYSIS

The advanced analysis should consider second order effect, initial imperfection of structures and members, the connection stiffness and material nonlinearity.

7.1. Computation Method

The three parameter computation method is used in the paper, including load control method, displacement control method and arc control method.

7.2. The Convergence of the Iterative Criterion

The relative error is controlled by the displacement norm [9].

$$\frac{\|\Delta\delta_i^j\|_2}{\|\delta_i^j\|_2} \le 10^{-6}$$

7.3. Structure Failure Criterion

The structure failure effect includes: loading model and geometric parameters of structure. When any one of these effects appears, the following may occur: the structure is failure: 1) three-dimensional plastic hinge is appeared in one element, 2) the number of plastic hinge is more than number of restrain, 3) the computation iteration does not converge, 4) the storey displacement is more than $h/50$.

8. EXAMPLES

Through advanced analysis, the NIDA software can consider initial imperfect state of structures and members, semi-rigid connections, geometry nonlinearity, and material nonlinearity. In design stage, the calculation length of the member is 1.0. The results by NIDA program are comparatively analyzed compared with the test data to verify availability of results from semi-rigid connections steel frame by advanced analysis.

The one span and two storey semi-rigid connections steel frame is shown in Fig. (**6**) below [8]. The section of beam and column is W5×16 , the steel grade is A36, yield strength is 248MPa, and the elastic modulus is 206850MPa. Firstly, the vertical load P=10.675kN is compressed at one-third length of beam at the first storey; and secondly, the horizontal load is compressed both in the first storey and the second storey. The value of load increase from zero to maximum value shows that the structure is a failure station.

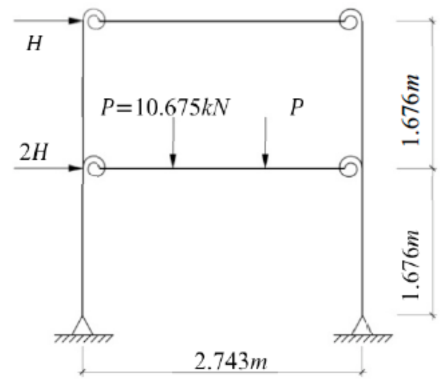

Fig. (6). Single span of two planes semi-rigid frame.

The column base is pin, the connection between beam and column is top and seat angle semi-rigid connections, and the section of angle is ∟4×4× 1/2.

The moment and rotation curve is simulated by three-parameter power model. The initial stiffness is $R_{ki} =$ 3374 kN·m/rad, the ultimate moment is $M_u = 20.9$ kN·m, and the shape coefficient is 1.65.

The results by NIDA program and the test data are shown in Fig. (**6**).

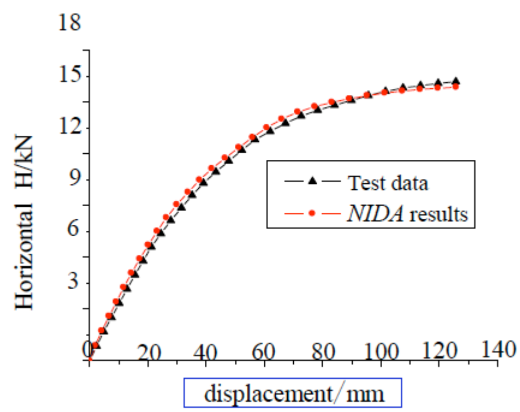

Fig. (7). Comparative analysis between NIDA results and test results.

When close to the limit load, the semi-rigid connections plastic hinge appears at the intersection of the first beam and column. When the horizontal load is increased, the semi-rigid connections plastic hinge appear at the second beam and column. The structure is the ultimate station, and the ultimate horizontal load is 16.3kN. The Fig. (**7**) shows that the results by NIDA program and the test data are close.

CONCLUSION

The advanced analysis method is introduced to analysis semi-rigid connections steel frame, considering initial imperfect mode of structures and members, geometric nonlinearity, material nonlinearity and semi-rigid connections. The results by the advanced analysis show the actual force of structures.

The results by NIDA program with advanced analysis method were comparatively analyzed with the test results data to verify availability.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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